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DEPARTMENT OF EDUCATION

MATHEMATICS

Grade 13

For Introduction in September, 1966

ISSUED BY AUTHORITY OF
THE MINISTER OF EDUCATION

REQUIREMENTS FOR

GRADE 13 MATHEMATICS FOR THE SCHOOL YEAR 1966-67

As indicated in the Introduction on page (i) of the course of study for Grade 13 Mathematics (Curriculum S.12C) each of Mathematics A and Mathematics B is to be shortened for the first years that the courses are in use.

This will be achieved for the school year 1966-67 by the deletions of specific topics in Mathematics A and by providing a choice of topics in Mathematics B.

Mathematics A

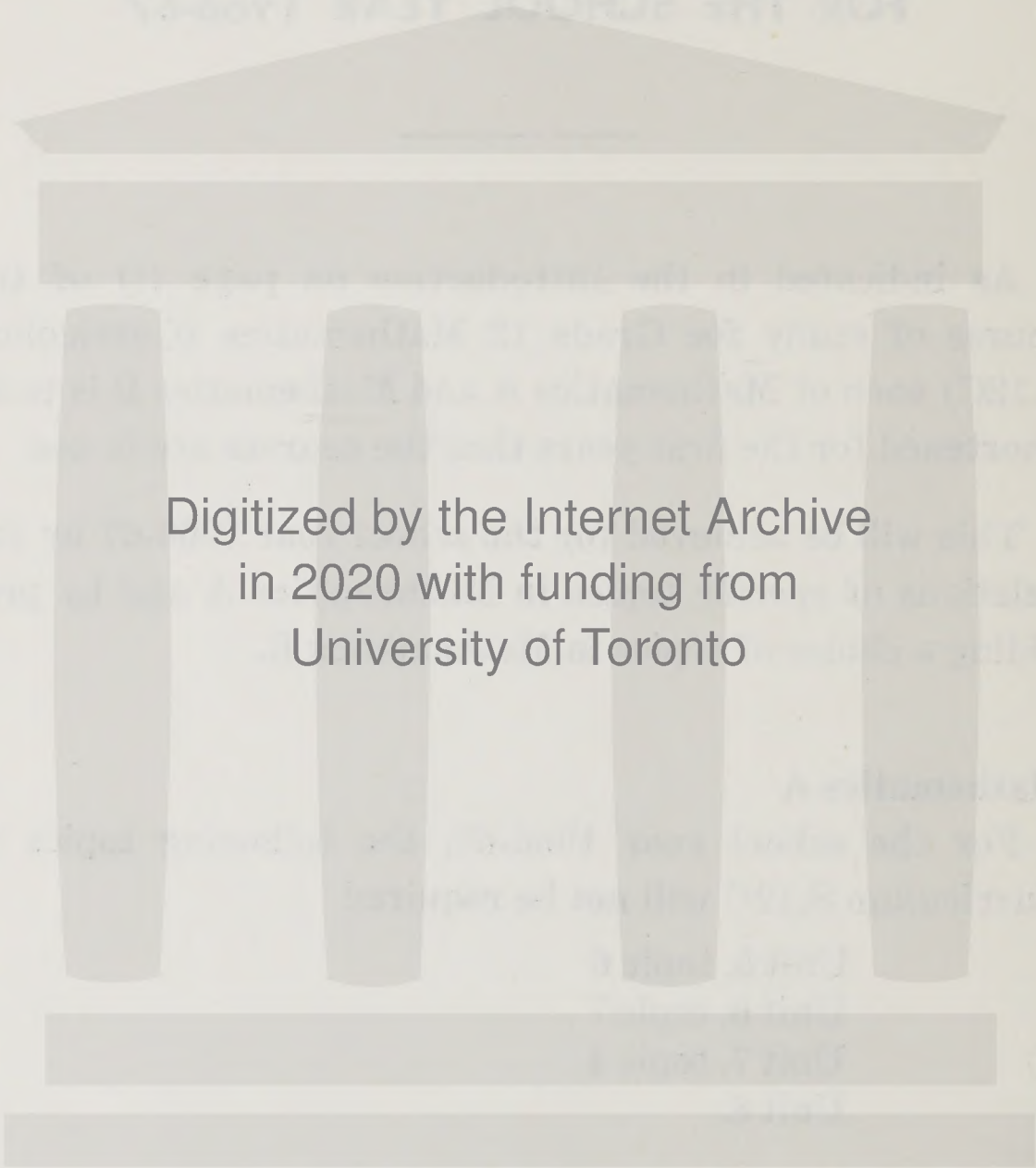
For the school year 1966-67, the following topics in Curriculum S.12C will **not** be required

Unit 5, topic 6
Unit 6, topic 7
Unit 7, topic 4
Unit 8.

Mathematics B

For the school year 1966-67, the course will consist of:

Units 1, 2, 4, 5, 6 and **at least two** of
Units 3, 7, 8, 9.



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MATHEMATICS

Grade 13

Introduction

1. Curriculum S.12C represents the third stage of a revision in mathematics of the Senior Division in the Five-Year Program. The first two stages are contained in Curriculum S.12A (Grade 11) and Curriculum S.12B (Grade 12) which were introduced in the schools in September 1964 and September 1965 respectively. The course outline of Curriculum S.12C will replace the courses in Grade 13 Algebra, Geometry, and Trigonometry and Statics outlined in Curriculum S.12 (1961).

2. The Grade 11 course is a study of the mathematics of the straight line. Grade 12 extends this work to a study of functions and the mathematics of the simpler curves. The Grade 13 course extends this study of functions with emphasis on the mapping concept which will be found recurring throughout to the extent that it may be considered the unifying concept of the course.

3. Curriculum S.12C provides a complete mathematics course for Grade 13 for introduction in September of 1966. It is designed for a total *instruction* time of nine hours per week in schools on the present organization. It is to be noted that nine hours per week corresponds to the present total instruction time for pupils who take all three mathematics subjects in Grade 13.

4. The course is organized in two parts, Mathematics A and Mathematics B, which are equal in planned instruction time but are somewhat different in type of content. This organization was adopted to allow the two parts to be taught in parallel and to allow freedom in assigning teachers in that it is not necessary for the same teacher to teach both Mathematics A and Mathematics B. While the present organization of the Grade 13 year continues, the two parts will be treated as two courses. Because of the nature of the subject matter, students

requiring only one Mathematics course will take Mathematics A. For this organization, eight periods per week should be allowed for Mathematics A and seven periods per week for Mathematics B.

5. The course outline includes considerable detail to indicate a possible method or order of development. It must be understood however that this detail is suggestive only, and that various orders of treatment are possible.

6. FOR THE FIRST YEARS THAT THIS COURSE IS IN USE IN THE SCHOOLS, EACH OF THE PARTS WILL BE SHORTENED BY THE DELETION OF TOPICS. THE TOPICS REQUIRED FOR EXAMINATION PURPOSES IN ANY GIVEN YEAR WILL BE ANNOUNCED BY DEPARTMENT OF EDUCATION MEMORANDA.

MATHEMATICS A

UNIT 1:

LOGICAL REASONING (1½ weeks)

This unit forms a short introduction to the general principles of proof (and disproof) in mathematics and to the vocabulary of logic.

1. Statements, sentences, open sentences, definitions

2. Logical connectives and quantifiers and their use in proof and disproof

UNIT 2:

FUNCTION AS A MAPPING (2 weeks)

The study of function begun in Grade 12 is pursued here with an emphasis on its interpretation as a mapping of one set into another.

1. Function defined as a mapping of the domain onto the range, in particular, real valued functions of a real variable.

   The notation $f:x \rightarrow f(x)$, that is, the function f that maps x onto $f(x)$

   Equivalence of this definition to the alternative definition in terms of ordered pairs

   One-to-one mappings

2. The inverse of a function

   The condition that the inverse of a function is also a function

   The relationship between the graph of a function and the graph of its inverse

   Practice in determining the domain and range of the inverse of a function

UNIT 3:
SECOND DEGREE RELATIONS IN THE PLANE
(6 weeks)

This unit extends the Grade 12 study of the second degree relations to develop the simpler properties of the conics.

1. A study of the graphs of relations defined by equations such as $x^2 + y^2 = r^2$, $y = ax^2$, $4x^2 + 9y^2 = 36$, $4x^2 - 9y^2 = 36$, $xy = 36$, $4x^2 - 9y^2 = 0$

   Graphing and naming of the corresponding curves and their relationship to the sections of a cone

   Determination in each case of domain, range, intercepts, and symmetry

   Study of the corresponding inequalities and their graphs

2. (a) Development of the equations of the parabola, ellipse, and hyperbola from the focus-directrix definitions

   (b) Development of the equations of the ellipse and hyperbola from the constant sum and constant difference definitions

   (c) Study of the equations of the conics in the other standard positions,

   $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, $\frac{x^2}{b^2}-\frac{y^2}{a^2}=-1$, $x^2-y^2=a^2$, $xy=k$, $y^2=4px$ and $x^2=4py$ (for positive and negative values of p)

3. Examples of the occurrence in applied mathematics of the parabola, the ellipse, and the hyperbola

4. Intersections of lines with conics

   Equations of tangents to particular conics determined by using the condition for equal roots

Regions defined by the conjunction or disjunction of simple linear-quadratic inequalities

5. Intersections of conics with conics

UNIT 4:
TRIGONOMETRY (4 weeks)

The Grade 12 course provides a study of the sine and cosine functions. This unit extends the study to the other four related functions and provides a development of the sum and difference formulas.

1. A review of the sine and cosine regarded as real valued functions of a real variable

   The definition of the related functions (tangent, cotangent, secant and cosecant) and a study of the domain, range and graph of each function

2. Development of the formulas for the sine, cosine and tangent of the sum and of the difference of two real numbers

   The trigonometric functions of $\pi \pm \theta$, $\frac{\pi}{2} \pm \theta$, $-\theta$

   Development of the formulas for $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$ in terms of functions of θ

   Some practice with trigonometric identities

3. A study of the phase shift, period and amplitude associated with $y = a \sin (k\theta + d)$ and $y = a \cos (k\theta + d)$

   Study of the sum $a \sin \theta + b \cos \theta$

4. Development of the expression for the angle between any two lines in terms of their slopes

UNIT 5:
TRANSFORMATIONS IN THE PLANE (5 weeks)

In this unit, translations, rotations, and reflections in the plane are studied as mappings. The transformations are then used to simplify second degree equations in order to identify the particular conics.

1. Translation developed as a mapping of the point (x, y) onto the point $(x + h, y + k)$

   Use of the notation $(x, y) \rightarrow (x + h, y + k)$

Graphical determination of images under translation of points, lines and simple polygons

Development that distance is invariant under translation

Observation that the measure of an angle is invariant under translation

Deduction from these facts that the image of the locus of any equation has the same geometric character as the given locus

Practice in determining the images under translation of the relations defined by

$$x^2+y^2=r^2,\ y^2=4px,\ \frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ \text{and}\ \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

Determination of the translation that will eliminate the first degree terms in equations of the form

$$x^2+y^2+2gx+2fy+c=0$$

2. Rotation developed as a mapping of the point (x, y) onto the point $(x\cos\theta - y\sin\theta,\ x\sin\theta + y\cos\theta)$ where θ denotes the measure of the counter-clockwise rotation of the *plane* about the origin.

   Graphical determination of the images under rotation of points, lines and simple polygons, where the angles of rotation are multiples of $\frac{\pi}{4}$ or $\frac{\pi}{6}$

   Verification for particular cases that distance and angles are preserved under rotation

3. Reflection (in the x or y axes) developed as a mapping of the point (x, y) onto $(-x, y)$ or $(x, -y)$

   Graphical determination of images under reflection of points, lines, and polygons

   Observation that distances and angles are invariant under reflection

4. Determination of the translation that will eliminate the first degree terms in equations of the form

   $$ax^2+by^2+2gx+2fy+c=0$$

   Intuitive determination of the form of the graph in various cases according as $ab=0$, $ab>0$, $ab<0$

5. Determination of the rotation that will eliminate the xy term in equations of the form

   $$ax^2+2hxy+by^2+2gx+2fy+c=0$$

6. Study of particular cases of the transformation
$$(x, y) \rightarrow (ax, by)$$
Observation that, when $a = b$, angles will be preserved, ratios of lengths of corresponding line segments will be preserved, but distances will not be preserved
Study of particular cases where $a \neq b$, to illustrate the distortion resulting from such transformations, e.g.,
$$(x,y) \rightarrow \left(\frac{x}{a}, \frac{y}{b}\right) \text{ in } x^2 + y^2 = 1$$

UNIT 6:
SLOPES AND SIMPLE DERIVATIVES (5½ weeks)

This unit begins an intuitive approach to calculus placing emphasis on the graphical and physical interpretation of the derivative, which in this treatment is defined as a new function f' such that $f'(x)$ equals the slope of the graph of the function at the point $(x, f(x))$.

1. An intuitive discussion of the limit of a sequence
The sum of an infinite geometric series
An intuitive approach to the definition of the limit of a function

2. Discussion of rate of change for a linear function
A review from Grade 11 of the slope of a straight line in terms of $\triangle x$ and $\triangle y$

3. Consideration of the problem of determining the (average) rate of change when other than linear functions are involved
Study of the graph of $y = x^2$ to determine the slopes of several secants, e.g., the one from $(2,4)$ to $(2 + \triangle x, 4 + \triangle y)$
A similar study of the function defined by $y = x^3$

4. Consideration of the problem of determining the instantaneous rate of change
A review from Grade 12 of the concept of tangent as the limiting position of secants
Application of this concept to the cases of the secants of $y = x^2$ and $y = x^3$ to determine the rate of change (the slope) at specified points

Checking of these values by measuring the slopes of the tangents at these points

Similar treatment at specified points on the graphs of equations such as $y = 2x^2 + 3x$, $y = x^3 - 9x$

5. Determination of the slope at the point (x_1, y_1) for several functions, e.g. the functions defined by

$$y = x^2,\ y = x^3,\ y = \frac{1}{x},\ y = 8x^3,\ y = 6,\ y = 2x^2 + 5x + 3$$

The definition of f': the derivative of f

The notations $D_x y$, $\frac{dy}{dx}$

6. Determination of the derivative of $x \to x^n$ where n is a positive integer or zero

Determination of the derivative with respect to x of a sum $u + v$, a product uv, and a quotient $\frac{u}{v}$, where u and v are differentiable functions of x

Determination of the derivative with respect to x of u^n where n is a positive integer and u is a differentiable function of x

Assumption of the derivative of u^r with respect to x where r is a rational number and u is a differentiable function of x

Practice in calculation of derivatives involving sums, products, quotients, and functions defined implicitly

7. The derivative of the functions $x \to \sin(kx + d)$ and $x \to \cos(kx + d)$

Problems in maximum and minimum values and in displacement, velocity and acceleration where trigonometric functions are involved

UNIT 7:

APPLICATIONS OF DIFFERENTIATION (5 weeks)

Many of the applications appearing in this unit will have been encountered by the pupils in previous experiences in mathematics and science. It is the objective of this unit to give practice with these practical problems.

1. Use of the derivative to find the slope of the tangent at a given point on a curve

Use of the derivative to find the equation of the tangent at a given point on a specific curve (particular cases only of circles, ellipses, parabolas and hyperbolas)

2. Problems determining average velocity during specified time intervals and the instantaneous velocity at specified times when given a relation between distance and time

   Calculation of $D_t s$ in various equations from physics involving s and t, e.g., $s = 16t^2$, $s = 200t - 16t^2$

   Interpretation of these derivatives

3. Consideration of the meaning of instantaneous change of velocity and its calculation as $D_t v$

   Problems in acceleration

4. Consideration of the graphical significance of the second derivative and the interpretation according as its value is positive, negative or zero

   Practice in curve-tracing using the first and second derivatives

5. A review from Grade 12 of the existence of an extreme value (maximum or minimum) of a quadratic function

   Determination of the extreme value using the first derivative

   Identification of the extreme value as either maximum or minimum

   Extension to other than quadratics

   Problems involving maxima and minima

6. Applications to rate problems

UNIT 8:
EQUATIONS OF THE FORM $D_x y = f(x)$ (4 weeks)

1. Solution by inspection of equations of the form $D_x y = f(x)$

   Finding the equation of a family of curves with a given slope

   Selection of a particular member of the family which satisfies a prescribed condition

2. Finding the velocity - time relation given the acceleration relation

Finding the distance - time relation given the velocity relation or the acceleration relation

Other problems involving distance, velocity, acceleration and time

3. Intuitive development, using a graph, to establish that the function f is the derivative of the area function A, and hence that A may be determined by solving the equation $A'(x) = f(x)$

   Calculation of the area bounded by a given curve and given ordinates

   Calculation by subtraction (or addition) of the area between two curves

MATHEMATICS B

UNIT 1:
SETS, SUBSETS, AND PERMUTATIONS (5 weeks)

1. Unions, intersections, and complements of sets

   Use of Venn diagrams to illustrate the commutative, associative, and distributive laws of intersection and union

   The complement of a union as the intersection of the complements

   The complement of an intersection as the union of the complements

2. Study of particular cases to establish the fundamental counting principles:
   (a) If a finite set A contains r elements and a finite set B contains s elements, then there are rs different ordered pairs (a, b) where $a \in$ A and $b \in$ B (or, equivalently, the Cartesian product $A \times B$ contains rs elements)
   (b) If a finite set A contains r elements, a finite set B contains s elements, and their intersection contains t elements, then the union of A and B contains $r + s - t$ elements

   Practice in determining the number of elements in $A \times B$, $A \cup B$, and $A \cap B$

   Extension to more than two sets

3. Definition of an r-arrangement (permutation) of n things as an ordered selection or arrangement of r of them

   The study of particular cases to establish that the number of r-arrangements of n things is

   $$n(n\text{-}1)(n\text{-}2)(n\text{-}3)\ldots(n-r+1)$$

   The particular case where $r=n$; the factorial notation, $n!$

   Number of arrangements of elements not all different

4. Definition of an r-subset (combination) of a set of n elements as a selection of r of them without regard to order

Development of the number of subsets of a set of n elements as 2^n

Development of the number of r-subsets of a set of n elements

Use of the notation $\binom{n}{r}$

UNIT 2:

MATHEMATICAL INDUCTION AND THE BINOMIAL THEOREM (3 weeks)

1. The method of mathematical induction

   Proof by mathematical induction of the sum of n terms of the general arithmetic and geometric series

   Use of examples to illustrate the properties of sigma notation for summation, namely

$$\sum_{i=1}^{n} 1 = n; \quad \sum_{i=1}^{n} a_i = \sum_{j=1}^{n} a_j; \quad \sum_{i=1}^{n} a(b_i) = a \sum_{i=1}^{n} b_i;$$

$$\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

   Use of mathematical induction to establish the formulas for the sum of the squares and of the cubes of the first n natural numbers

   Study of a few examples proved by mathematical induction and by some other method

   Study of at least one example which is true for $n = 1, 2, 3, \ldots, k$ and is false for $n = k + 1$

2. The proof of the binomial theorem for positive integral exponents by the use of the number of r-subsets

   Statement of the general term in the binomial expansion

   Reference to the use of Pascal's triangle

   The proof of the binomial theorem by mathematical induction

UNIT 3:
PROBABILITY (3 weeks)

This unit is designed to provide an introduction to the topic of probability. The development of the topic is restricted to situations that yield only a finite number of possible outcomes. Solutions of problems may be by the use of tree diagrams or by the use of the sample space.

1. The definition of the probability P of an event where the total number of outcomes is finite
   Practice in writing the set of all possible outcomes
   Practice in calculating the probability of an event
   Interpretation of situations indicated by $P=0$ and by $P=1$
   Discussion of odds
   Binomial Distribution

2. Use of a Venn diagram to illustrate two events, A and B, that are not necessarily mutually exclusive
   Development of the probability that A or B will occur, that is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
   Derivation of $P(A \cap B) = 0$ and of $P(A \cup B \cup C) = P(A) + P(B) + P(C)$, where A, B, and C are mutually exclusive events

UNIT 4:
VECTORS (7 weeks)

The first sections of this unit are designed to provide an introduction to vectors without the use of a coordinate system. In each of the sections of this introduction it is intended that there will be frequent use of illustrations and examples for motivation and for clarification of the concepts involved. Sections 8, 9, 10, 11, 12, 13 introduce coordinate systems for the study of vectors. Sections 14, 15, and 16 introduce the measurements associated with vectors.

1. Vectors as directed line segments in 2-space and in 3-space *without* a coordinate system
2. Equality of vectors with attention to the fact that only length and direction are relevant

3. Addition of vectors by use of parallelograms and triangles

   Subtraction of vectors

   The negative of a vector

   The zero vector

   Multiplication by a scalar

4. The algebraic properties of vector addition and multiplication by a scalar illustrated by geometrical constructions

5. Linear combinations of vectors

   The conditions that (i) two vectors be collinear, (ii) three vectors be coplanar and (iii) three vectors have their end points on a line (internal and external division of a line segment)

6. Use of vectors to prove some propositions from deductive geometry, e.g.,

   (a) If two sides of a quadrilateral are equal and parallel, the other two sides are also equal and parallel

   (b) The diagonals of a parallelogram bisect each other

   (c) The line joining the mid-points of two sides of a triangle is parallel to the third side and equal in measure to one-half of it

   (d) The medians of a triangle intersect at a point at two-thirds of the distance from each vertex

7. Resultant and equilibrant of forces acting at a point

   Resultant of velocities

8. The Cartesian coordinate system for 2 and 3 dimensions

   Identification of points with pairs of triples

9. Definition of addition, subtraction, and multiplication by a scalar for pairs or triples

10. Verification that pairs and triples satisfy the algebraic properties of vector addition and multiplication by a scalar

    Geometric interpretation of triples (pairs) as vectors

11. Lines and planes through the origin as linear combinations of given vectors
    Collinear and coplanar vectors

12. Demonstration that, for any three vectors in 2-space, at least one is a linear combination of the other two
    Development that, for any two non-collinear vectors in 2-space, any vector is a linear combination of them, and extension of this concept to non-coplanar vectors in 3-space

13. The standard bases (1, 0), (0, 1) and (1, 0, 0), (0, 1, 0), (0, 0, 1)

14. The distance formula and length of line segments in two and in three dimensions

15. The length of a vector, $|\mathbf{a}|$
    Dot or inner product $\mathbf{a} \cdot \mathbf{b}$
    Relation to the cosine law
    The algebraic properties of dot product

16. Angles and projections
    Perpendicularity
    Unit vectors and normalization

17. Applications to resolution of forces and work

UNIT 5:
EQUATIONS OF LINES AND PLANES (3 weeks)

1. Lines and planes in general position

2. The vector equation of a line in 2-space
   The resulting parametric equations of a line in 2-space
   The linear equation of a line in 2-space (developed using the inner product)

3. The vector equation of a line in 3-space
   The resulting parametric equations of a line in 3-space
   Direction angles, cosines and numbers

4. The vector equation of a plane in 3-space
   The resulting parametric equations of a plane in 3-space

The linear equation of a plane in 3-space (developed using the inner product)

5. Geometric interpretation of the solution set of two or of three linear equations

UNIT 6:
SYSTEMS OF LINEAR EQUATIONS (2½ weeks)

1. Solution of m equations (homogeneous and non-homogeneous) in n unknowns by eliminating variables (specific values of m and n not too large)
   Inconsistent systems
2. Definition of the augmented matrix of a system of m equations in n unknowns
   Development of the technique of obtaining a row reduced echelon form of the augmented matrix
3. The solution in parametric form, or the recognition of inconsistency, as read from a row reduced echelon form of the augmented matrix
4. For $n \leq 3$, interpretation of these parametric solutions as lines or planes as described in Unit 5
5. Discussion of the $m=n=3$ case, consistent or inconsistent, in terms of the intersections of planes, by considering the normal vectors in addition to the foregoing techniques

UNIT 7:
MATRICES AND LINEAR TRANSFORMATIONS
(3 weeks)

In this unit, matrices and the application of matrices to linear transformations are introduced. While the definitions are general, practice with the operations and applications is to be restricted to cases where the number of rows and columns is two.

1. Definition of an m by n matrix, $A=(a_{ij})$, (m, n arbitrary but not too large)
   Row and column vectors
   Definition of Ax where x is n by 1
2. Linear equations of the form $Ax=b$

3. Ax as a function of x
   Statement and proof of linear properties
4. Examples: rotations, reflections, projections, shears, identity, zero
5. Product AB defined as composition
   The rule $AB = \left(\sum_{j=1}^{n} a_{ij} b_{jk} \right)$
   Observation that AB generalizes Ax, and that the dot product is a matrix multiplication
6. Associativity, non-commutativity, identity and zero divisors
7. Definition of A^{-1} and its calculation
   Interpretation of A^{-1} as the matrix of the inverse of the linear transformation with matrix A
   Non-invertible matrices

UNIT 8:
COMPLEX NUMBERS AND POLAR COORDINATES
(3 weeks)

1. Complex numbers as ordered pairs and in the form $a+bi$
2. Field properties of the complex numbers
3. Solutions of quadratic equations
4. Geometric form of complex numbers
   Absolute value
   Complex conjugates
5. Polar form of complex numbers
6. Establishment of the correspondence for the point P between the rectangular coordinates (x, y), the polar coordinates (r, θ), and the rectangular components of the vector OP
   Practice at conversions from any one of these pairs to the other two
   Practice in graphing of relations expressed in polar form, particularly $r=a$, $r=a \cos \theta$, $r=a \sin \theta$, $r=a \sec \theta$, $r=a \operatorname{cosec} \theta$, $r=a+b \cos \theta$, $r=a \cos 2\theta$, $r=a \sin 3\theta$, $r=a\theta$

7. (Supplementary) de Moivre's Theorem
   Applications to n'th roots and multiple angle formula

UNIT 9:
EXAMPLES OF GROUPS (3 weeks)

1. Definition of group as a set closed under a binary operation that possesses both an inverse and identity (neutral) element

2. Demonstration that translations, rotations and reflections form groups
   Use of two-dimensional models of regular polygons of 3, 4, 5, 6 sides to establish the group characteristics associated with these transformations

3. Examination of the systems of natural numbers, integers, rational numbers, real numbers and complex numbers to determine which systems are groups

4. Examination of sets of functions that form groups ($f \circ g$ defined as $f(g)$ and use of f^{-1} as the group inverse)
   Permutation developed as a one-to-one mapping of a set onto itself
   Non-commutative groups

5. The representation of rotations and reflections in the plane by means of 2×2 matrices
   Direction of attention to the closure property
   Groups of matrices

6. Identification of the characteristic properties of rings and fields
   Examples of rings and fields

7. (Supplementary) Transformations of a regular tetrahedron and cube